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ELEMENTARY ABSORPTION OF LIGHT PASSING THROUGH CLOUDS

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GEOPHYSICS: ELEMENTARY ABSORPTION OF LIGHT PASSING THROUGH CLOUDS[1]
[Geofisica: Assorbimento elementare della luce nel passagio attraverso alle nubi]

Notes of Guglielmo Zanotelli, presented by
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In the theory of the propagation of light through clouds, developed by Mecke [1], Albrecht [2], and Gordov [3], the properties of the medium are represented by two coefficients which express the disposition of the element of a cloud, that is, a drop of water, to diffuse and absorb light, respectively.

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[2]Received at the Academy July 30, 1940

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In the present work, I propose to demonstrate how the second of the two mentioned coefficients can be expressed as a function of the physical characteristics of the water drop, determining theoretically for light however diffused and polarized which penetrates this water drop, the quantity which will be absorbed and dissipated.

I will follow the procedures of geometrical optics apart from the consideration of each phenomenon of difraction, whose influence is, on the other hand, negligible when considering water drops of a sufficiently large diameter with respect to the wavelength of the light.

We will begin by observing that diffuse light reaches a certain water drop, in which we intend to study the absorption and which we will call for simplicity an illuminated drop, from the surrounding drops which are found at varying distances from it. Here we propose first of all to investigate for how many of such drops the portion of the surface of the light wave emitted by whichever point of one of them, intersected by the surface of the illuminated drop, can be considered plane.

Let us suppose therefore that $\mathfrak{N}$ is the number of drops contained in a unit volume of a cloud and that they are all of equal radius R. Thus $4\pi l^2 \mathfrak{N}\, dl$ drops are obviously contained in the interval between two spheres with the center at the center O of the illuminated drop and radii l and $l + dl$. Each one of these drops is seen from O under a solid angle which is approximately given by $\pi R^2/l^2$; that is, the drops contained in said interval collectively intercept a portion of the solid angle $4\pi^2 \mathfrak{N} R^2 dl$, which, as we shall see, is independent of l. Supposing for simplicity that a water drop diffuses light equally in all directions, it can be said that the quantity of light which collectively reaches the illuminated drop

from drops placed in a certain layer at whatever determined distance is proportional to said solid angle and therefore is always equal whether they are close to or far away from O. However, it must be observed that there is a certain maximum distance L beyond which the drops which are outside of the sphere of radius L cease to send their light to O since the entire solid angle 4π is already completely intercepted; therefore:

$$4\pi = \int_0^L 4\pi^2 \mathfrak{N} R^2 \, dl, \qquad L = \frac{1}{\pi \mathfrak{N} R^2}.$$

It must be observed that the radius of the sphere which contains all of the drops which exert some influence on O is actually greater than L since the defense action that a drop in a certain layer could exert on a drop in a layer closer to O is not taken into account. This action obviously corresponds to a reduction in the intercepted solid angle and in conclusion, an increase in O. However, such a correction, which could be deduced by considerations of probability, is not important for our purpose, which is that of determining the maximum aliquot of water drops contained in the aforementioned sphere, for which the waves emitted by them at any point S, and intersected by the illuminated drop's surface, cannot be considered plane.

Such an aliquot will be constituted by all of the water drops close enough to O to be contained in a sphere of the radius $L_o < L$, where L_o is the minimum value for which it can be acknowledged that the magnitude of R/L_o is neglibible in comparison with 1, thus setting the condition that the portion of the spherical wave emitted from S and intersected by the surface of the

illuminated drop can conform to a plane. Such a condition being considered verified to our standards when $R/L_o = 0.1$, we intend now to determine L_o in such a manner that it is a small fraction of L. If one assumes $L_o = 0.01$ L, this is the same as saying that under the preceeding considerations only 1% of the diffuse radiation which inclusively reaches O originates in the drops contained inside the sphere of radius L_o, that is, that of this radiation, 99% can be considered as incident on the illuminated drop with plane wave surfaces.

It follows therefore that: $L = 100\ L_o = 1000\ R = 1/\pi R^2 \mathfrak{N}$, therefore $\pi R^3 \mathfrak{N} = 1/1000$. Using Q to indicate the quantity of water contained in a cubic centimeter, we have: $Q = 4\pi R^3 \mathfrak{N}/3 = 1.33 \times 10^{-3}$ g/cm^3. It can therefore be concluded that if no more than one milligram of water per cubic centimeter, in round figures, is contained in clouds, it can be considered with certainty that of the light which reaches whatever water drop by diffusion from the surrounding drops, no more that 1% is made up of light originating in water drops close enough so that the portion of the surface of the wave intersected by the surface of the illuminated drop cannot be considered plane.

Let us investigate, therefore, the case of a water drop illuminated by the light diffused by the surrounding drops at distances great enough so that the portion of the surface of the wave intersected by this drop can be considered plane. We will determine for each surface of the wave the percentage of the incident light absorbed and dissipated inside of the illuminated drop. The noted percentage turns out to the independent of the direction of propagation and of the polarization state of the incident light, dependent only on the characteristic of the illuminated drop, and which is,

nevertheless, equal for all surfaces of the wave: it coincides therefore with the coefficient of absorption sought.

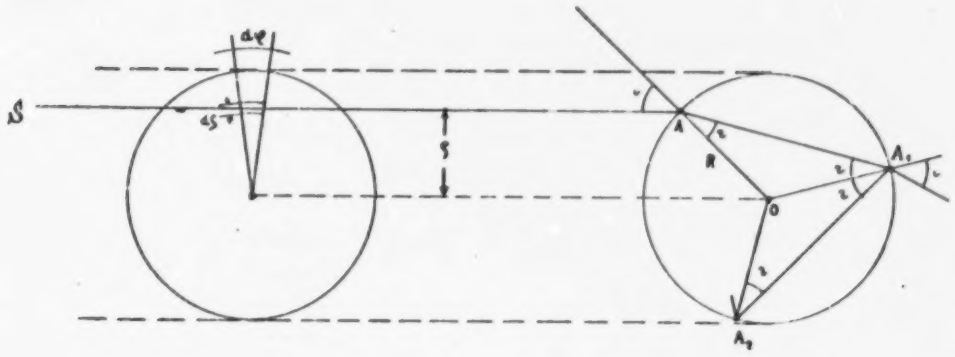

Fig. 1.

Consider a spherical drop of water of radius R and a light ray SA (Fig. 1) incident at the point A on the surface of the drop under angle i: it is assumed that the plane passing through the radius and through the center O of the water drop, that is the plane of incidence, is the plane in the figure; and P and N are the intensities of the components of the electrical luminous field, in a plane and normally, respectively. The angle of refraction, r, is obviously equal to the angle of incidence, A_1, of the ray refracted in the first refraction inside of the water drop, and to the successive angles of incidence and reflection. In the refraction at A, part of the incident light passes in the reflected ray while the intensities of the two components, normal and parallel, of the refracted ray can be obtained by means of the formulas of Fresnel [4] and are respectively:

$$N_0 = N\left[\frac{2\sin r\cos i}{\sin(i+r)}\right]^2 \quad , \quad P_0 = P\left[\frac{2\sin r\cos i}{\sin(i+r)\cos(i-r)}\right]^2.$$

In the passage from A to A_1 there is an attenuation of the refracted ray according to the law of absorption, and calling the coefficient of absorption of water α, being $AA_1 = 2R \cos r$ at the arrival at A_1, the above-mentioned intensity will have become:

$$N_0' = N_0 e^{-2\alpha R \cos r} = N \left[\frac{2 \sin r \cos i}{\sin (i + r)} \right]^2 e^{-2\alpha R \cos r},$$

$$P_0' = P_0 e^{-2\alpha R \cos r} = P \left[\frac{2 \sin r \cos i}{\sin (i + r) \cos (i - r)} \right]^2 e^{-2\alpha R \cos r}.$$

In the first reflection at A_1, because part of the light is refracted outside, there is a subsequent attenuation calculable by the formula of Fresnel, and for the reflected ray the intensity is precisely:

$$N_1 = N_0' \left[\frac{\sin (r - i)}{\sin (r + i)} \right]^2 = N \left[\frac{2 \sin r \cos i}{\sin (i + r)} \right]^2 \left[\frac{\sin (r - i)}{\sin (r + i)} \right]^2 e^{-2\alpha R \cos r},$$

$$P_1 = P_0' \left[\frac{\tan (r - i)}{\tan (r + i)} \right]^2 = P \left[\frac{2 \sin r \cos i}{\sin (i + r) \cos (i - r)} \right]^2 \left[\frac{\tan (r - i)}{\tan (r + i)} \right]^2 e^{-2\alpha R \cos r}.$$

In the passage from A_1 to A_2, there is another attenuation by absorption, and in A_2 an attenuation in the second reflection in a manner such that the intensity will have become:

$$N_2 = N \left[\frac{2 \sin r \cos i}{\sin (i + r)} \right]^2 \left[\frac{\sin (r - i)}{\sin (r + i)} \right]^4 e^{-4\alpha R \cos r},$$

$$P_2 = P \left[\frac{2 \sin r \cos i}{\sin (i + r) \cos (i - r)} \right]^2 \left[\frac{\tan (r - i)}{\tan (r + i)} \right]^4 e^{-4\alpha R \cos r},$$

and in general, after k reflections:

$$N_k = N \left[\frac{2 \sin r \cos i}{\sin (i + r)} \right]^2 \left[\frac{\sin (r - i)}{\sin (r + i)} e^{-\alpha R \cos r} \right]^{2k},$$

$$P_k = P \left[\frac{2 \sin r \cos i}{\sin (i + r) \cos (i - r)} \right]^2 \left[\frac{\tan (r - i)}{\tan (r + i)} e^{-\alpha R \cos r} \right]^{2k}.$$

The luminous intensity absorbed in the passage from A_k to A_{k+1} for the two components is clearly given by:

$$N_k(1 - e^{-2\alpha R\cos r}) \quad , \quad P_k(1 - e^{-2\alpha R\cos r})$$

and the intensity collectively absorbed in the passage of the light inside of the water drop is respectively:

$$N_a = \sum_0^\infty{}_k N_k(1 - e^{-2\alpha R\cos r}) =$$

$$= N\left[\frac{2\sin r\cos i}{\sin(i+r)}\right]^2 (1 - e^{-2\alpha R\cos r}) \sum_0^\infty{}_k \left[\frac{\sin(r-i)}{\sin(r+i)} e^{-\alpha R\cos r}\right]^{2k},$$

$$P_a = \sum_0^\infty{}_k P_k(1 - e^{-2\alpha R\cos r}) =$$

$$= P\left[\frac{2\sin r\cos i}{\sin(i+r)\cos(i-r)}\right]^2 (1 - e^{-2\alpha R\cos r}) \sum_0^\infty{}_k \left[\frac{\tan(r-i)}{\tan(r+i)} e^{-\alpha R\cos r}\right]^{2k}.$$

It is now necessary to consider the two functions:

$$u(i) = \frac{\sin(r-i)}{\sin(r+i)} \quad , \quad v(i) = \frac{\tan(r-i)}{\tan(r+i)}$$

which can be constructed when the index of refraction n in water is known, this being sin i = n sin r; assuming an approximate value n = 4/3, the two functions are represented by the graph in Fig. 2, where the values of the functions for i = 0° to 90° are given. It follows from the graph that in said interval the absolute values of u and v are always less than 1, and become equal to 1 for i = 90°. On the other hand, for $\alpha \neq 0$, the magnitude of $x = 2\alpha R\cos r$ in the same interval is always positive and not zero, and therefore, the magnitudes of u^2e^{-x} and v^2e^{-x} are either zero or positive and less than 1. Therefore the two series contained in the preceding formulas under summation signs converge and can be expressed as sums of geometric progressions. Therefore:

$$N_a = N\left[\frac{2\sin r\cos i}{\sin(i+r)}\right]^2 \frac{1-e^{-x}}{1-u^2e^{-x}},$$

$$P_a = P\left[\frac{2\sin r\cos i}{\sin(i+r)\cos(i-r)}\right]^2 \frac{1-e^{-x}}{1-v^2e^{-x}}.$$

The absorption coefficient of water α has, for radiation between 4500 Å and 8100 Å, a very small value which reaches, according to the data of Aschkinass [5], a maximum of 0.0241 cm^{-1} corresponding to the wave lengths between 7500 Å and 7750 Å. Considering R always less than 0.25 cm [6], since cos r reaches a value of about 0.66 corresponding to i = 90°, x is always less than or equal to 8×10^{-3}. It can therefore be assumed in such approximations that $1 - e^{-x}$ is equal to x, and substituting in the preceding formulas, one has with each successive formal transformation:

$$N_a = N\left[\frac{2\sin r\cos i}{\sin(i+r)}\right]^2 \frac{x}{1-u^2+u^2x} = \frac{2\alpha RN}{n}\, p(i)\cos i,$$

$$P_a = P\left[\frac{2\sin r\cos i}{\sin(i+r)\cos(i-r)}\right]^2 \frac{x}{1-v^2+v^2x} = \frac{2\alpha RP}{n}\, q(i)\cos i$$

where it is assumed that:

$$p(i) = \frac{1}{1+\frac{u^2x}{1-u^2}}, \quad q(i) = \frac{1}{1+\frac{v^2x}{1-v^2}}.$$

The two functions N_a and P_a have similar behavior, which coincides to a large extent in the interval between 0° and 90° with the behavior of cos i, since for small values of x, the functions p(i) and q(i) remain essentially equal to 1 approaching the vicinity of i = 90°, where they tend to zero. Fig. 3 represents the behavior of p(i) and q(i) for the value $x = 8 \times 10^{-3}$, and with the scale used, the curves of the two functions are essentially superimposed.

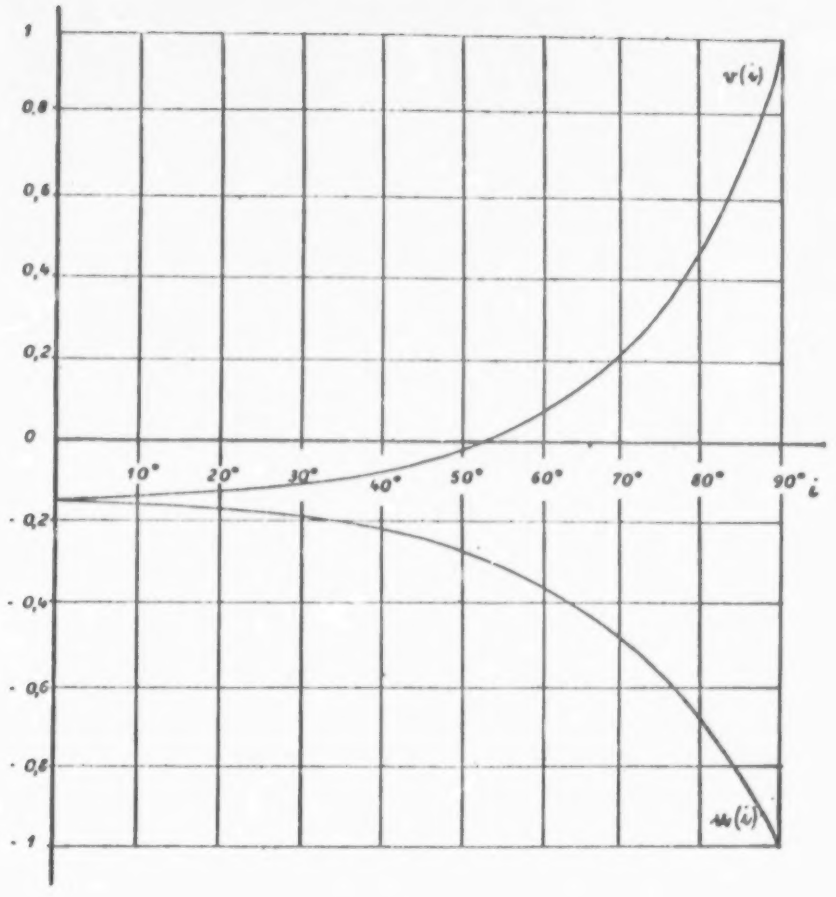

Fig. 2.

The above-mentioned formulas furnish the luminous intensity absorbed inside of the water drop as a function of the angle of incidence, therefore it will be necessary to sum the contributions to absorption by rays incident at all angles between 0° and 90° in order to know the total energy absorbed when the water drop is invested with a bundle of parallel light. To this end, a straight line section of the bundle which enters the drop will be considered, and in it, an element of the surface itself constituted by a portion of the circular corona of radius ρ and thickness $d\rho$ (Fig.1)

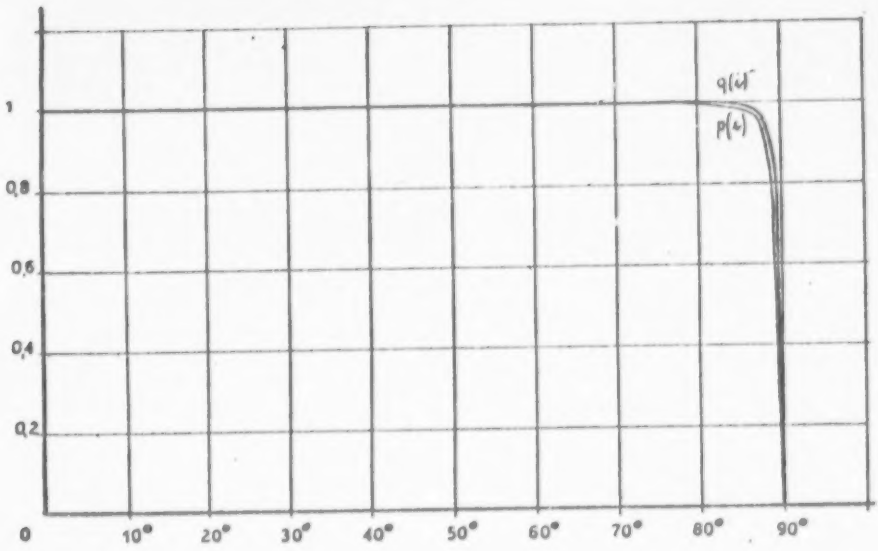

Fig. 3.

included between two rays making between them the elementary angle $d\phi$: the area of said element is then given by $\rho\, d\phi d\rho$, that is, since $\rho = R \sin i$, by $R^2 \sin i \cos i\, di\, d\phi$: the luminous potency which crosses such an area is for each of the components N and P expressed by:

$$dW_n = N R^2 \sin i \cos i\, di\, d\varphi ,$$
$$dW_p = P R^2 \sin i \cos i\, di\, d\varphi ;$$

The part of such a potency which will be absorbed in the water drop is instead, applying the formulas found:

$$dW_{an} = N_a R^2 \sin i \cos i\, di\, d\varphi = \frac{2\alpha R^3 N}{n} p(i) \sin i \cos^2 i\, di\, d\varphi ,$$

$$dW_{ap} = P_a R^2 \sin i \cos i\, di\, d\varphi = \frac{2\alpha R^3 P}{n} q(i) \sin i \cos^2 i\, di\, d\varphi .$$

In order to proceed with the integration, we observe that the functions p(i) and q(i) remain essentially equal to 1 approaching the vicinity of i = 90° where, on the contrary, the contribution to the integral by the integrated function is small enough because of the presence of the factor $\cos^2 i$ which is almostnull there. We will of course disregard, therefore, in the integration, the two factors p(i) and q(i), except to take account of the error thus introduced into the result.

With this approximation, the total energy absorbed is, indicating by J the intensity of the incident light:

$$dW_a = \frac{2\alpha R^3 (N + P)}{n} \sin i \cos^2 i \, di \, d\varphi = \frac{2\alpha R^3 J}{n} \sin i \cos^2 i \, di \, d\varphi$$

which does not depend on the polarization state of the latter.

Extending the integration to all section of the bundle, therefore:

$$W_a = \frac{2\alpha R^3 J}{n} \int_0^{2\pi} d\varphi \int_0^{\pi/2} \sin i \cos^2 i \, di = \frac{4\pi\alpha R^3 J}{3n} .$$

Taking into account the presence of the functions p(i) and q(i), the exact value obtained, W'_a, was a little smaller as a consequence of the fact that in the proximity of i = 90° both p(i) and q(i) tend to zero, and therefore the integrated function also tended to zero somewhat more rapidly there. In order to discuss the result, we therefore observe that for the maximum assumed value $x = 8 \times 10^{-3}$, and for i = 78°, the two above-mentioned functions assume values which differ from 1 by about 0.5% and in the least 0.2%, respectively, therefore with good reason, we can leave out, in th approximation to 1%, the integrated function for the full interval from 0° to 78°. In the interval from 78° to 90° the contribution which the integrated function makes to the magnitude of W'_a, which, by the fact that both

p(i) and q(i) tend to zero in the vicinity of 90°, is somewhat less than:

$$\Delta W_a = \frac{4\pi\alpha R^3 J}{n} \int_{78^\circ}^{90^\circ} \sin i \cos^2 i \, di = 8.99 \cdot 10^{-3} \frac{4\pi\alpha R^3 J}{3n} .$$

The exact value W_a' of the absorbed energy is thus such that $W_a > W_a' > W_a - \Delta W_a$, and consequently, the value W_a of our determination is a rounded-up value of less than 1%, being precisely:

$$\frac{\Delta W_a}{W_a} = 8.99 \cdot 10^{-3} < \frac{1}{100} .$$

Because the potency which passes through the section of the bundle and hits the water drop is $W = \pi R^2 J$, the percentage which will be absorbed is given by:

$$x = \frac{W_a}{W} = \frac{4\alpha R}{3n} .$$

Since it has been shown that 99% of the light waves which reach the drop from surrounding water drops are plane (under the premise that no more than one milligram of water is contained in a cubic centimeter of a cloud); and since for the remaining 1% of the light waves which are not plane, only a slight correction for the percentage absorbed is foreseen (the path of the light inside the water drop remaining on the same order of magnitude), it may be concluded that:

The value $x = 4\alpha R/3n$ gives the coefficient of absorption, that is, the relationship between incident light and absorbed light, within 1%,

in a spherical water drop of radius R contained in a cloud and illuminated by light however diffuse and of wavelengths in the band between 4500 Å and 8100 Å.

Using an approximate value for n of 4/3, substituting in the formula found, we have $x = \alpha R$, which agrees with the value $4\alpha R/3$ presented by Albrecht in the work cited, showing that this value is exactly 1/3 greater than the real value, and this is due to the fact that having taken the index of refraction equal to 1, he did not take into account the light which does not effectively penetrate the water drop due to reflection on entrance, and therefore which is not subject to absorption.

We will consider as infinite the absorption in an element of unit volume in the cloud in which $\mathfrak{N}$ water drops are contained, all of the radius R, and which is invested by a plane wave of intensity J. Using $4\pi R^3 \alpha J/3n$ as the energy absorbed in each water drop, the energy absorbed in the unit of volume is given by:

$$W_{ta} = \frac{4\pi R^3 \alpha J \mathfrak{N}}{3n} = \frac{Q \alpha J}{n}$$

where Q indicates the quantity of water contained in the cloud per unit of volume. Thus, the result in this case is:

The absorbed energy, the volume of the cloud remaining equal, is independent of the light ray and of the number of water drops, and depends only on the total quantity of water contained, that is, increase or decrease in absorption corresponds only to condesation or evaporation, respectively.

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